

Sampling quantum phase space with squeezed states

Konrad Banaszek* and Krzysztof Wódkiewicz

*Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, Hoża 69,
PL-00-681 Warszawa, Poland*

**E-mail address: Konrad.Banaszek@fuw.edu.pl*

Abstract: We study the application of squeezed states in a quantum optical scheme for direct sampling of the phase space by photon counting. We prove that the detection setup with a squeezed coherent probe field is equivalent to the probing of the squeezed signal field with a coherent state. An example of the Schrödinger cat state measurement shows that the use of squeezed states allows one to detect clearly the interference between distinct phase space components despite losses through the unused output port of the setup.

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1. Introduction

Phase space quasidistribution functions are a convenient way of characterizing the quantum state of optical radiation [1]. Over past several years, they have gained experimental significance due to the reconstruction of the Wigner function of a single light mode performed using tomographic algorithms [2]. Recently, an alternative method for measuring quasidistribution functions of a light mode has been proposed [3,4]. The method is based on photon counting of the signal field superposed on a probe field in a coherent state. The advantage of this method is that there is no complicated numerical processing of

the experimental data. A simple arithmetic operation performed on the photocount statistics yields directly the value of a quasidistribution function at a point defined by the amplitude and the phase of the coherent field.

The purpose of this communication is to study the application of squeezed states in the proposed photon counting scheme. The most important feature of squeezed states is that quantum fluctuations in some observables are reduced below the coherent state level [5]. In the context of optical homodyne tomography, the squeezing transformation has been shown to be capable of compensating for the deleterious effect of low detection efficiency [6]. Therefore, it is interesting to discuss the information on the quantum state of light which can be retrieved in a photon counting experiment using squeezed states.

2. Experimental scheme

We start with a brief description of the proposed setup, depicted in Fig. 1. The field incident on a photodetector is a combination, performed using a beam splitter with a power transmission T , of a transmitted signal mode and a reflected probe mode. The statistics of the detector counts $\{p_n\}$ is used to calculate an alternating series $\sum_{n=0}^{\infty} (-1)^n p_n$. In terms of the outgoing mode, this series is given by the expectation value of the parity operator:

$$\hat{\Pi} = (-1)^{\hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}}, \quad (1)$$

where the annihilation operator of the outgoing mode \hat{a}_{out} is a linear combination of the signal and the probe field operators:

$$\hat{a}_{\text{out}} = \sqrt{T} \hat{a}_S - \sqrt{1-T} \hat{a}_P. \quad (2)$$

The expectation value of the measured observable involves statistical properties of both the signal and the probe modes. The operator $\hat{\Pi}$ can be written in the following normally ordered form:

$$\hat{\Pi} = : \exp[-2(\sqrt{T} \hat{a}_S^\dagger - \sqrt{1-T} \hat{a}_P^\dagger)(\sqrt{T} \hat{a}_S - \sqrt{1-T} \hat{a}_P)] : , \quad (3)$$

which has a clear and intuitive interpretation within the Wigner function formalism: the measured quantity is proportional to the phase space integral of the product of the signal and the probe Wigner functions with relatively rescaled parameterizations [4]. Hence the proposed scheme is a realization of direct sampling of the quantum phase space.

An important class of probe fields are coherent states $\hat{a}_S|\alpha\rangle_P = \alpha|\alpha\rangle_P$. The quantum expectation value over the probe mode can be easily evaluated in this case using the normally ordered form given in Eq. (3). Thus the measured observable is given by the following operator acting in the Hilbert space of the signal mode:

$$\langle \alpha | \hat{\Pi} | \alpha \rangle_P = : \exp[-2(\sqrt{T} \hat{a}_S^\dagger - \sqrt{1-T} \alpha^*)(\sqrt{T} \hat{a}_S - \sqrt{1-T} \alpha)] : . \quad (4)$$

This observable is closely related to a certain quasidistribution function. The most straightforward way to identify this link is to recall that an s -ordered quasidistribution function at a complex phase space point β is given by the expectation value of the normally ordered operator:

$$\hat{U}(\beta; s) = \frac{2}{\pi(1-s)} : \exp \left[-\frac{2}{1-s} (\hat{a}_S^\dagger - \beta^*)(\hat{a}_S - \beta) \right] : . \quad (5)$$

After a simple rearrangement of parameters we finally arrive at the formula:

$$\langle \alpha | \hat{\Pi} | \alpha \rangle_P = \frac{\pi}{2T} \hat{U} \left(\sqrt{\frac{1-T}{T}} \alpha; -\frac{1-T}{T} \right). \quad (6)$$

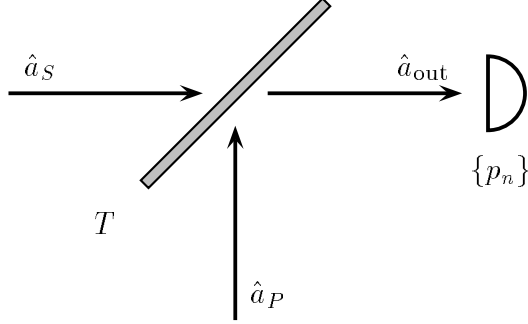


Fig. 1. The setup for direct probing of the quantum phase space. The detector measures the photocount statistics $\{p_n\}$ of a signal \hat{a}_S combined with a probe field \hat{a}_P using a beam splitter with a power transmission T .

Thus, the alternating series computed from the photocount statistics yields the value of a quasidistribution function at a point $\sqrt{(1-T)/T}\alpha$ defined by the amplitude and the phase of the probe coherent field. The complete quasidistribution function can be scanned point-by-point by changing the probe field parameters.

The ordering of the measured quasidistribution function depends on the beam splitter transmission. This is a consequence of the fact that a fraction of the signal field escapes through the second unused output port of the beam splitter. These losses of the field lower the ordering of the detected observable. This effect is analogous to the one appearing in balanced homodyne detection with imperfect detectors [7,8]. In the limit $T \rightarrow 1$, when the complete signal field is detected, we measure directly the Wigner function, corresponding to the symmetric ordering.

3. Sampling with squeezed state

We will now consider the case when a squeezed coherent state $S_P(r, \varphi)|\alpha\rangle_P$ enters through the probe port of the beam splitter. We use the following definition of the squeezing operator for an i th mode:

$$S_i(r, \varphi) = \exp[r(e^{-i\varphi}\hat{a}_i^2 - e^{i\varphi}(\hat{a}_i^\dagger)^2)/2]. \quad (7)$$

The detected quantity is now given by the expectation value of the following operator acting in the Hilbert space of the signal mode:

$$\langle \hat{\Pi} \rangle_P = \langle \alpha | \hat{S}_P^\dagger(r, \varphi) \hat{\Pi} \hat{S}_P(r, \varphi) | \alpha \rangle_P. \quad (8)$$

In order to find an interpretation for this observable, we will derive a formula for the squeezing transformations of the parity operator $\hat{\Pi}$. We start from a simple unitary transformation:

$$(-1)^{\hat{a}_{out}^\dagger \hat{a}_{out}} \hat{a}_{out}^2 (-1)^{\hat{a}_{out}^\dagger \hat{a}_{out}} = e^{i\pi \hat{a}_{out}^\dagger \hat{a}_{out}} \hat{a}_{out}^2 e^{-i\pi \hat{a}_{out}^\dagger \hat{a}_{out}} = e^{-2\pi i} \hat{a}_{out}^2 = \hat{a}_{out}^2. \quad (9)$$

This equation implies the commutator:

$$[(-1)^{\hat{a}_{out}^\dagger \hat{a}_{out}}, e^{i\varphi}(\hat{a}_{out}^\dagger)^2 - e^{-i\varphi}\hat{a}_{out}^2] = 0, \quad (10)$$

which states that generation or annihilation of pairs of photons conserves parity. Therefore, the parity operator is invariant under the squeezing transformation:

$$\hat{S}_{\text{out}}^\dagger(r, \varphi) \hat{\Pi} \hat{S}_{\text{out}}(r, \varphi) = \hat{\Pi}. \quad (11)$$

This identity has nontrivial consequences when written in terms of the signal and the probe modes. It is equivalent to the equation:

$$\hat{S}_S^\dagger(r, \varphi) \hat{S}_P^\dagger(r, \varphi) \hat{\Pi} \hat{S}_P(r, \varphi) \hat{S}_S(r, \varphi) = \hat{\Pi} \quad (12)$$

which, after moving the signal squeezing operators to the right hand side, yields the following result:

$$\hat{S}_P^\dagger(r, \varphi) \hat{\Pi} \hat{S}_P(r, \varphi) = \hat{S}_S^\dagger(-r, \varphi) \hat{\Pi} \hat{S}_S(-r, \varphi) \quad (13)$$

This formula shows that squeezing of the probe mode is equivalent to squeezing of the signal mode with the opposite sign of the parameter r . This change of the sign swaps the field quadratures that get squeezed or antisqueezed under the squeezing transformation.

Finally we obtain the following explicit expression for the detected signal field observable:

$$\begin{aligned} \langle \hat{\Pi} \rangle_P &= \hat{S}_S^\dagger(-r, \varphi) \langle \alpha | \hat{\Pi} | \alpha \rangle_P \hat{S}_S(-r, \varphi) \\ &= \frac{\pi}{2T} \hat{S}_S^\dagger(-r, \varphi) \hat{U} \left(\sqrt{\frac{1-T}{T}} \alpha; -\frac{1-T}{T} \right) \hat{S}_S(-r, \varphi). \end{aligned} \quad (14)$$

Thus, the setup delivers again an $s = -(1-T)/T$ -ordered quasidistribution function at a phase space point $\sqrt{(1-T)/T}$, but corresponding to a *squeezed* signal field.

Let us note that it was possible to carry the squeezing transformation from the probe to the signal degree of freedom only due to a specific form of the measured observable. We have explicitly used the conservation of the parity operator during generation or annihilation of pairs of photons. For a general observable defined for the outgoing mode \hat{a}_{out} , there is no formula analogous to Eq. (13).

4. Detection of Schrödinger cat state

As an illustration, we will consider a photon counting experiment for a Schrödinger cat state, which is a quantum superposition of two coherent states [9]:

$$|\psi\rangle = \frac{|i\kappa\rangle + |-i\kappa\rangle}{\sqrt{2 + 2\exp(-2\kappa^2)}}, \quad (15)$$

where κ is a real parameter. The Wigner function of such a state contains, in addition to two positive peaks corresponding to the coherent states, an oscillating term originating from quantum interference between the classical-like components. This nonclassical feature is extremely fragile, and disappears very quickly in the presence of dissipation [10].

As we have found in Eq. (14), the outcome of the photon counting experiment with a squeezed probe field is related to an s -ordered quasidistribution of the squeezed Schrödinger cat state $\hat{S}_S(-r, \varphi)|\psi\rangle$. For simplicity, we will restrict ourselves to the case $\varphi = 0$. A simple but lengthy calculation yields the explicit formula for the phase space quasidistribution at a complex point $\beta = q + ip$:

$$\begin{aligned} &\langle \psi | \hat{S}_S^\dagger(-r, 0) \hat{U}(q + ip; s) \hat{S}_S(-r, 0) | \psi \rangle \\ &= \frac{\exp\left(-\frac{2q^2}{e^{2r} - s}\right)}{\pi[1 + \exp(-2\kappa^2)]\sqrt{1 - 2s \cosh 2r + s^2}} \left\{ \exp\left[-\frac{2(p - e^{-r}\kappa)^2}{e^{-2r} - s}\right] \right\} \end{aligned}$$

$$+ \exp \left[-\frac{2(p + e^{-r}\kappa)^2}{e^{-2r} - s} \right] + 2 \exp \left(\frac{2s\kappa^2}{e^{2r} - s} - \frac{2p^2}{e^{-2r} - s} \right) \cos \left(\frac{4e^r\kappa q}{e^{2r} - s} \right) \Big\}. \quad (16)$$

In Fig. 2 we depict the expectation value of the parity operator $\langle \hat{\Pi} \rangle$ as a function of the rescaled complex probe field amplitude $\beta = \sqrt{(1-T)/T}\alpha$. For comparison, we show two cases: when the Schrödinger cat state is probed with coherent states $|\alpha\rangle_P$ and squeezed coherent states $\hat{S}_P(r=1,0)|\alpha\rangle_P$. The beam splitter transmission is $T = 80\%$. When coherent states are used, only a faint trace of the oscillatory pattern can be noticed due to losses of the signal field. In contrast, probing of the Schrödinger cat state with suitably chosen squeezed states yields a clear picture of quantum coherence between distinct phase space components. This effect is particularly surprising if we realize that 20% of the signal field power is lost through the unused output port of the beam splitter.

The visibility of the oscillatory pattern depends substantially on the sign of the squeezing parameter r . This can be most easily understood using the Wigner phase space description of the discussed scheme [4]. In order to detect the interference, fluctuations in the probe squeezed states have to be reduced in the direction corresponding to the rapid oscillations of the Wigner function corresponding to the Schrödinger cat state. The width of the rescaled probe Wigner function along the squeezed direction must be smaller than the spacing between the interference fringes.

5. Conclusions

We have studied the quantum optical scheme for direct sampling of the quantum phase space using squeezed coherent states. We have shown that squeezing transformations performed on the signal and the probe input ports of the setup are equivalent. The application of squeezed states with the appropriately chosen squeezing direction allows one to detect quantum interference despite losses through the unused output port of the setup.

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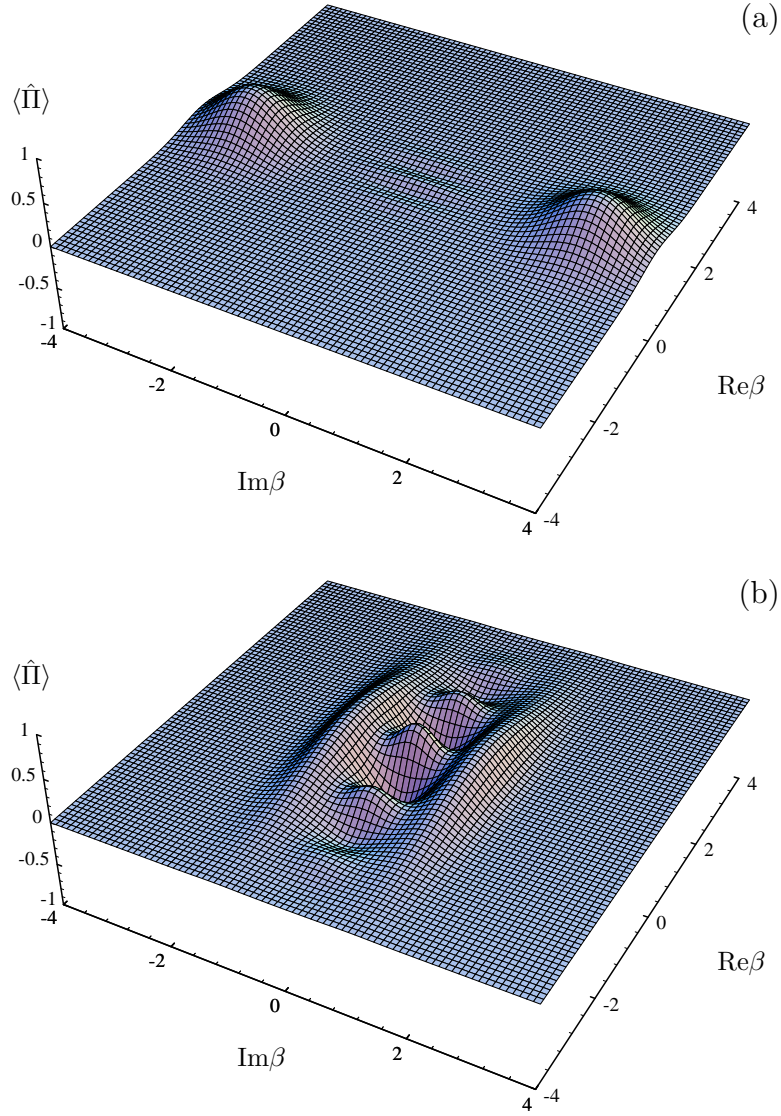


Fig. 2. Sampling the Schrödinger cat state $|\psi\rangle \propto |3i\rangle + |-3i\rangle$ with: (a) coherent states $|\alpha\rangle_P$ and (b) squeezed states $\hat{S}_P(r=1,0)|\alpha\rangle_P$. The plots show the expectation value of the parity operator $\langle \hat{\Pi} \rangle$ as a function of the rescaled complex probe field amplitude $\beta = \sqrt{(1-T)/T}\alpha$. The beam splitter transmission is $T = 80\%$.